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Visuals and Vocabulary:
The Next Generation in Mathematics Education

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Visuals and Vocabulary:
The Next Generation in Mathematics Education

by

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Report

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Dedication

The following research is completely and whole-heartedly dedicated: To my parents, Mr. & Mrs. Elias and Yolanda R. Olivarez, without whom my life would have failed to be interesting long ago; To my teachers, without whom my love of learning would have burned out; To my students, without whom my love of teaching would have ceased before it began; And lastly, to my best friends, without whom my time during this process would have been much harder.

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Abstract

Visuals and Vocabulary:
The Next Generation in Mathematics Education

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In recent years, there has been a growth of using visuals and vocabulary in mathematics and mathematics classes. The purpose of this Master's Report is to illuminate research done in the realm of mathematics education related to the increasing use of visuals and visual devices as models for mathematical concepts, as well as visuals for quick reference or "short cuts." Also discussed is mathematics vocabulary, the words most likely seen on mathematics exams, standardized state tests, and overall, any vocabulary most likely to trigger problem solving strategies and solutions.

Trends such as "word walls" and "graphic organizers," as well as vocabulary strategies aimed at oral, visual, and kinesthetic learners have all emerged in the classroom. Other strategies implemented and researched include student mathematics

journals, student created mathematics dictionaries, children's literature, graphic organizers, and written explanations of open ended word problems. All proved to enhance students' mathematical vocabulary, increase comprehension and increase ability in communication of mathematical ideas. Furthermore, the use of visual models has emerged in mathematics courses in order to promote more mathematical understanding. "Proofs without words" and patterns and pictures are growing in their use to explain mathematical concepts and ideas. Visual devices that help students arrive at probable answers also have grown in their implementation in the classroom and beyond.

Overall, has the increased use of visuals and vocabulary in both mathematics education and mathematics in general improved the mathematical understanding of our society? What research, if any, has been done to document the effects of word walls, graphic organizers, and etcetera? The research will show that, yes, an overwhelming amount of data shows that the implementation of such visual and vocabulary strategies can improve the mathematical understanding of those exposed to the strategies and devices.

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INTRODUCTION

Why Visuals & Vocabulary?

Many first year mathematics teachers are immediately thrown into a world they have been preparing for in their teacher certification programs, but still do not fully understand. The author, much like many new secondary mathematics teachers, dreamed of being a teacher, and started the adventure of mathematics education by teaching Algebra 1, the basis of all secondary mathematics courses. The author was ready with lesson plans in hand. However, the shock comes with the hard realization that teachers are not going to be teaching just their specific branch of mathematics, but also English, Spanish, and everything in between. Many new teachers have to learn quickly that they are not just "Mathematics teachers", but the bridge between daily "conversation language" and academic "mathematics language." Those teachers who cannot adapt to a new style of teaching fast enough are often the teachers that fall into the group that leave the teaching career early. What the author learned in the first few years of her teaching career is detailed in this Report. It's safe to say that dreams can come with lesson plans of their own.

DESCRIPTION OF MAIN TOPIC

In recent years, there has been a growth of using visuals and vocabulary in mathematics and mathematics classes. The purpose of this Master's Report is to illuminate research done in the realm of mathematics education related to the increasing use of visuals and visual devices as models for mathematical concepts, as well as visuals for quick reference or "short cuts." Also discussed is mathematics vocabulary, the words most likely seen on mathematics exams, standardized state tests, and overall, any vocabulary most likely to trigger problem solving strategies and solutions. Trends such as "word walls" and "graphic organizers" have been made mandatory by educational administrators for some mathematics teachers. Many, like Thompson and Rubenstein, suggest several strategies to promote mathematical vocabulary, including oral, visual, and kinesthetic methods to promote vocabulary growth [**16**, p. 2-5].

The use of visual models has emerged in mathematics courses in order to promote more mathematical understanding. "Proofs without words" are becoming popular as mathematicians strive to present mathematical theorems in relatable environments. In other words, when the mathematics becomes centered in real life situations, it is considered easier to understand, and more applicable to the people studying the mathematics. Patterns and proofs of mathematics concepts in different ways all aim to increase the mathematical vocabulary and visual mind. What used to be long paragraphs of explanations and justifications can now be compressed into a picture or pattern easier to comprehend.

Vocabulary lessons and activities have grown to help students and mathematicians retain mathematical concepts. Tools such as word walls function as conversational and visual scaffolds that help students take control of literacy skills and strategies [1, p. 700]. After an action research project conducted by Blessman and Myszczack, student mathematics journals, student created mathematics dictionaries, children's literature, graphic organizers, and written explanations of open ended word problems all proved to also enhance students' mathematical vocabulary, increase comprehension and increase ability in communication of mathematical ideas [**3**, p. 1].

Has the increased use of visuals and vocabulary in both mathematics education and mathematics in general improved the mathematical understanding of our society?

What research, if any, has been done to document the effects of word walls, graphic organizers, and etcetera? Furthermore, if no documentation is available to warrant the use of these purely visual proofs, then what are we teaching future generations of mathematicians?

Reasons

The author spent the first year teaching Algebra at Sydney Lanier High School in Austin, Texas. The mostly economically disadvantaged student population of Lanier High School, was much like the low performing high school the author had attended just four years earlier. Over 70% of the student population of Lanier qualified for free or reduced lunch, labeling the school as Title 1 under the Elementary and Secondary Education Act. The excitement that usually comes with first year teaching quickly faded when the author discovered she would be teaching Algebra 1 for English as a Second Language Learners. By definition, English as a Second Language Learners are students whose primary language at home is not English. Furthermore, a coteacher would be available daily to help with the students that could not speak or read any English. Not having had any training in teaching English Language Learners, and not having been certified to teach English as a Second Language courses, the author/teacher, much like most first year teachers, was thrown into several days of professional development workshops and trainings aimed at teaching the teacher several strategies utilizing visuals and vocabulary. Over the next two years, the author/teacher underwent several observations from administrators and appraisers purely to check such things as having a word wall available in the classroom, and/or if the teacher was using the strategies with which she had been presented. The new teacher was commended by educational administrators on the use of vocabulary in the classroom, but still did not see the growth in the students' test scores as the teacher would have liked or expected. Like most teachers, the author was always told, "It's alright. They're ESL." However, most teachers would not take that as an excuse. During the author's third year of teaching, the author/teacher was given a much higher performing group of students, but still expected to use the same visual and vocabulary strategies from the previous two years. Like several other mathematics teachers at the author's school, the author/teacher felt as though they were limiting the mathematics development by replacing it with pictures. When the staff was given the book, "Do I Really Have to Teach English?," the author/teacher, began to question the requirement of such basic graphic organizers and wanted to know if any documented growth due to the use of these strategies, since her experience had not yielded any results. This Master's Report is the result of a quest for such documentation.

Chapter 1: Vocabulary & Visuals in Mathematics Courses

VOCABULARY IN MATHEMATICS COURSES

In a mathematics classroom or mathematics course, vocabulary is the use of mathematics specific words and definitions, not necessarily used or understood in daily conversation (for example, words such as “slope”, “perpendicular”, “matrix”, and “function”). If asked for a list of student weaknesses, teachers in middle school are likely to say limited vocabulary, basic mathematical properties, and knowledge of the world [**5**, p. 528]. Word walls, considered as “print-rich”, are one of the tools utilized to inspire the literacy development of mathematics students. Most word walls share common characteristics, such as the words are selected for specific instructional purposes and collections are cumulative [**1,** p. 700]. As some have also felt, “word walls can be effective literacy tools that hold the potential for enhancing vocabulary learning when used in conjunction with effective instructional practices” [**17**, p. 398]. However, if students are not actively engaged with the word walls, learning is obstructed [**10**, p. 398]. Bradham and Villaume suggest using the word walls daily with beginner, developing, and struggling readers and writers. The words on the wall then become anchored in long term memory in ways that allow quick reference and promote detection of patterns and connections between words [**1**, p. 700]. The use of word walls for older students, such as those in middle school, then would hopefully inspire the interest in learning new words and enable these students to relate them to what they are learning in their math classes [**10**, p. 399]. Harmon would argue that educators know that the environment influences the learning of the students, and more specifically, that the classroom environment influences literacy development [**10**, p. 398]. The word wall becomes more than words on the wall and instead becomes an interactive tool that can facilitate discussions, expand students' vocabulary, and engage learning.

With such great potential, one must question if word walls actually work? Furthermore, what measure of effectiveness would be considered appropriate when referencing word walls and their success rate? In a study of 44 seventh graders, 63% of whom were labeled as white, all the students stated that the word wall with pictures and colors was more useful than a word wall with words alone [**10**, p. 405]. These colorful examples could be as simple as using different colored chalk to represent congruent sides of triangles, to more elaborate drawings showing a specific activity [**11**, p. 154]. Students were recorded as saying that the word walls with colors and pictures helped them to recall hints as to the meaning and even the use of the word in real life situations. This level of thinking suggests the interactive word wall sparked not just basic memorization, but higher level thinking and problem solving. Furthermore, one student remembered and used the mental image of the word wall on a state standardized assessment, even though the wall had been covered up [**10**, p. 406].

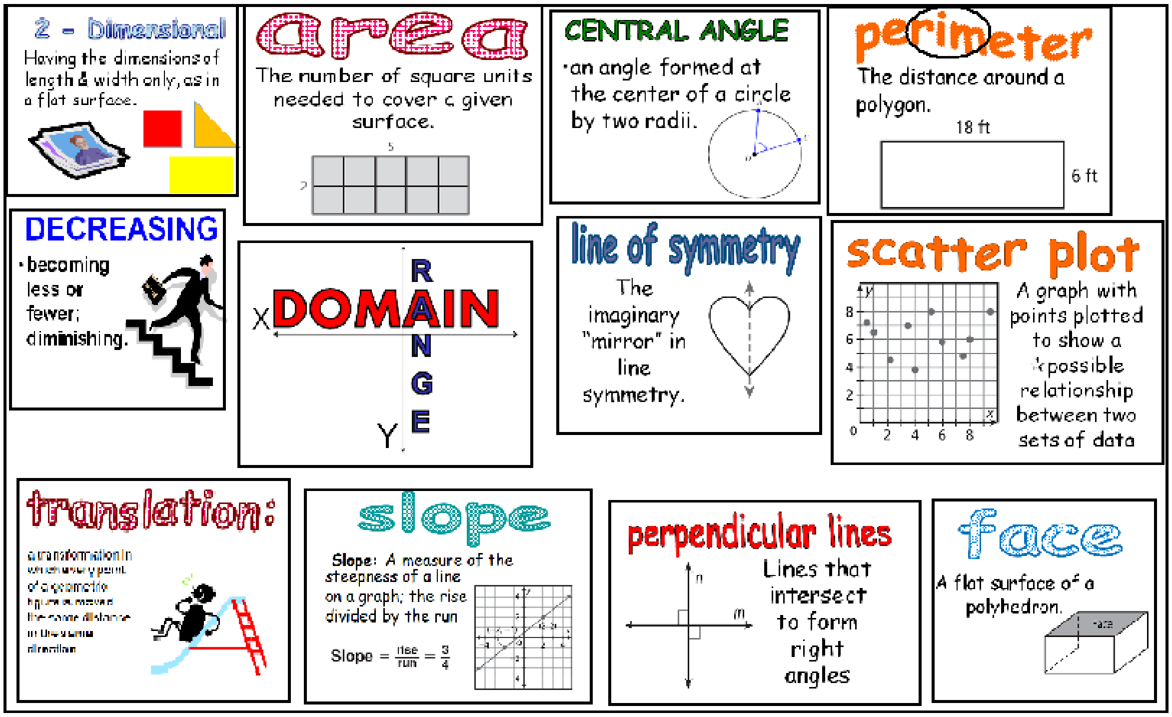

Figure 1. Example of Word Wall with Color Found in Author's Classroom

An overwhelming number of the students commented on how much they enjoyed the interactive lesson activities to create the word wall and, therefore, enjoyed using the wall since it was a product of their own making [**10**, p. 406]. This reminds educators that the word wall, in order to be effective, must be more than vocabulary words made visible by being displayed on a classroom wall. Since 70% of the most frequently used words have multiple meanings, pictures and examples help students to narrow down the exact use of the word [**5**, p. 531]. This is in accordance with Bromley, who states that students have a better chance of remembering vocabulary if it is stored by being linked to some schema or network of organized information [**5**, p. 531]. Unfortunately, when results of six weekly teacher developed assessments were compared between a word wall classroom, and a non word wall classroom, no significant differences in the students' scores were found. However, when an assessment was given to the students in both classrooms without warning several weeks later, the students in the word wall classroom performed much higher on the assessment than students in the non word wall classroom based on their scores. Students in the word wall classroom showed a more long term deeper level of understanding of the vocabulary and mathematics learned. This is a goal of vocabulary instruction [**10**, p. 406].

VISUALS IN MATHEMATICS COURSES

Visuals and visual aids in mathematics courses stand to serve the same purpose as vocabulary strategies; to enhance the mathematical understanding of all students and mathematicians. In another study composed by Douthitt, visuals in the form of slides accompanied with synchronized narration was expected to better meet the needs of students in both reading and mathematics, as opposed to more traditional lecture type instruction [**12**, p. 183]. A control group of students had lectures three times a week, while a laboratory group had lectures twice a week and a lab session once a week, arranged in small groups of five or six students and a mathematics tutor. Within the laboratory group, half the students used the textbook as a primary resource and the other half used models, graphic organizers, and other visual aids [**12**, p. 184]. While Douthitt found significant differences between the control group and laboratory groups, there were no significant differences between the laboratory groups themselves [**12**, p. 184]. This would suggest there was no difference caused by the types of audio or visual aids used. Upon further investigation it appeared the mathematics tutors involved had some affect on the students' learning as well.

Visual aids also serve to help the student connect mathematical information or to arrive at a certain numeric answer. The visual devices used can be as simple as a box of circles and polygons, each representing a different property, or four rulers bolted together to form a loose jointed parallelogram [**11**, p. 155]. Other visual aids having been used in secondary mathematics courses include the 4 corner model for representing linear and quadratic functions, mnemonic devices for memorizing the order of operations such as "Please excuse my dear Aunt Sally," and the "fish method" when solving mathematical proportions. Furthermore, in a study conducted by Caglayan and Olive, eighth grade students used cups and tiles to represent solving and writing equations in one unknown [**6**, p. 143]. Examples of these visual tools are displayed below:

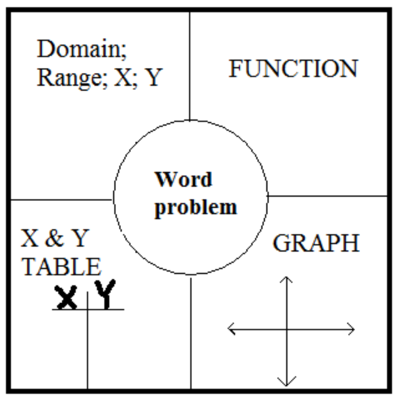

Figure 2. Example of 4 Corner Model for Linear Functions Used in Author's Classroom

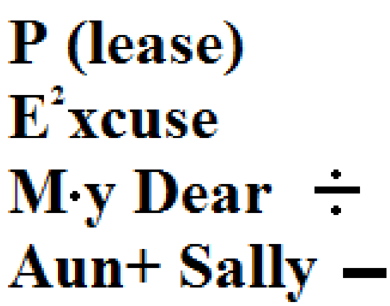

Figure 3. Example of Visual Mnemonic Used in Author's Classroom

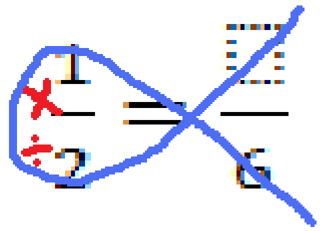

Figure 4. Example of Fish Method Used in Author's Classroom

Hoag also offers visual devices that allow students to take on a more hands on approach. Having secondary mathematics students reduce the solar system to scale not only brings a sense of personal accomplishment, but allows the students to explore large distances in relation to everyday life. Discussing how a pantograph works to bring forward geometric principles, visiting a construction site to see what methods were being implemented to make a home level, and using walking sticks to predict shadows are all visual methods to implementing mathematics [**11**, p. 158].

Chapter 2: Visuals & Vocabulary in Secondary Mathematics

VISUALS IN SECONDARY MATHEMATICS

According to Borwein and Jorgension, French mathematician Jacques Hadamard was of the opinion that mathematical thought is visual and that words only interfered [**4**, p. 897]. Borwein and Jorgensen would argue the same, taking note that some of the most influential leaps in mathematics have been visual, such as Descartes' "Cartesian" coordinate plane. There is no surprise then that in accordance with the growth of visuals and vocabulary in secondary mathematics courses, there has also been a shift in higher level mathematics as well, with more "proof without words" popping up in the mathematics world. Borwein and Jorgensen would credit this shift in large part to the technology era, since "new computational environments have greatly increased the scope of visualizing mathematics." Whatever the cause, the "visual proof" has emerged. For the non professional, the idea that mathematicians "see" their ideas may be surprising [**4**, p. 897]. However, in order to more deeply relate to the mathematics seen in daily life, mathematicians are forced to reinvent theorems and proofs, putting aside for a moment equations and two column justifications in exchange for pictures that are easier for society to understand and to recognize. In "visual proofs," however, the path through the information is usually indeterminate, leaving the viewer to determine what is important and how the information is connected [**4**, p. 899].

For example, an *octagonal number* is defined as a number that creates an octagon and subsequent patterns of octagons [**14**, p. 200]. Originally, octagonal numbers were accepted as simply a pattern that could be derived with a simple formula, namely

$$3n^2 - 2n, \ with\, n > 0.$$

However, as previously stated, times are changing and now a more visual approach to octagonal numbers has emerged. More specifically, octagonal numbers can be visually represented as the difference of two squares [**14**, p. 200].

$$1 = 1 = 1^2 - 0^2$$
$$1 + 7 = 8 = 3^2 - 1^2$$
$$1 + 7 + 13 + 19 = 40 = 7^2 - 3^2$$
$$O_n = 1 + 7 + \cdots + (6n - 5) = (2n - 1)^2 - (n - 1)^2$$
$$O_n = 1 + 7 + \cdots + (6n - 5) = 3n^2 - 2n$$

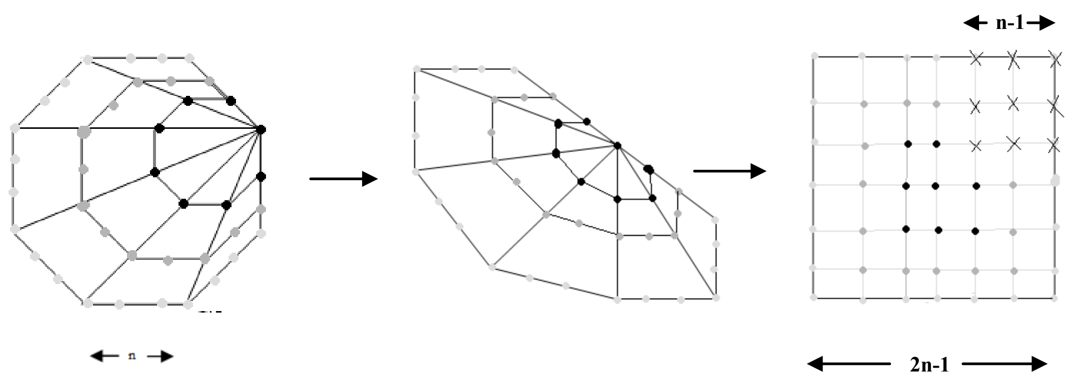

Figure 5. Visual Proof of Octagonal Numbers as the Difference of Two Squares [**14**, p. 200]

As demonstrated previously, the mathematics has evolved from a formula to a visual representation that students and mathematicians alike can recognize and manipulate to remember exactly what constitutes an octagonal number and its properties [**14**, p. 200]. However, it is up to the viewer of the proof to establish that the figure is changing from an octagon into a large square with a small square being "subtracted" from its area (also known as the "difference" of two squares).

There is no end to the visual world, since even the Pythagorean Theorem and Pythagorean Triples can be "proven visually" as having a one-to-one correspondence to the factorizations of even squares of the form

$$p^2 = 2nm.$$

Gomez begins by stating the following assumptions or "givens;"

If $a^2 + b^2 = c^2$, then $(a + b - c)^2 = 2(c - 1)(c - b)$.

If $p^2 = 2nm$, then $(p + n)^2 + (p + m)^2 = (p + n + m)^2$.

The Pythagorean Theorem, often discussed and modeled in secondary mathematics classes as a right triangle with squares attached to both legs and hypotenuse, is then modeled as in fact, a much smaller, easier picture to comprehend and understand [**8**, p. 14].

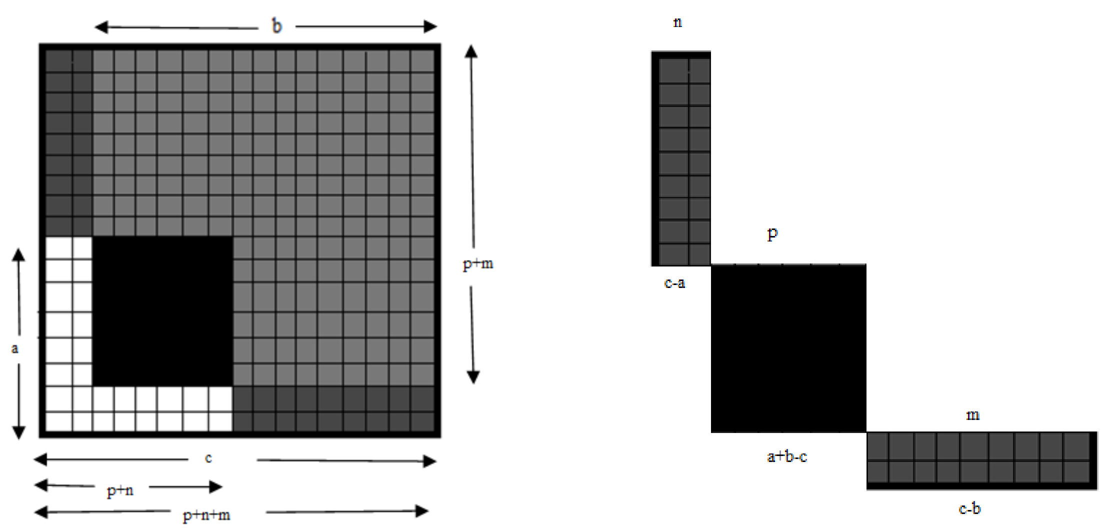

Figure 6. Pythagorean Theorem as Factorization of Even Squares [**8**, p. 14]

As demonstrated above, the Pythagorean Theorem is redefined and reinvented into a simple picture that reminds the mathematician looking at the "proof without words" that a timeless truth can easily be demonstrated visually. Borwein and Jorgensen warn the viewer of the visual proof to check the correctness of the implementation. In fact, visual proof viewers should inspect a visual representation with the same rigor as a traditional logical formal proof [**4**, p. 900].

Much higher level mathematics, including number theory, has also seen a growth in visuals and mathematical vocabulary usage. Number Theory, which identifies patterns of relationships between numbers, is a required course in most college and university mathematics degree programs [**4**, p. 900]. Complex long proofs can now be summarized in short, heuristic diagrams such as the following:

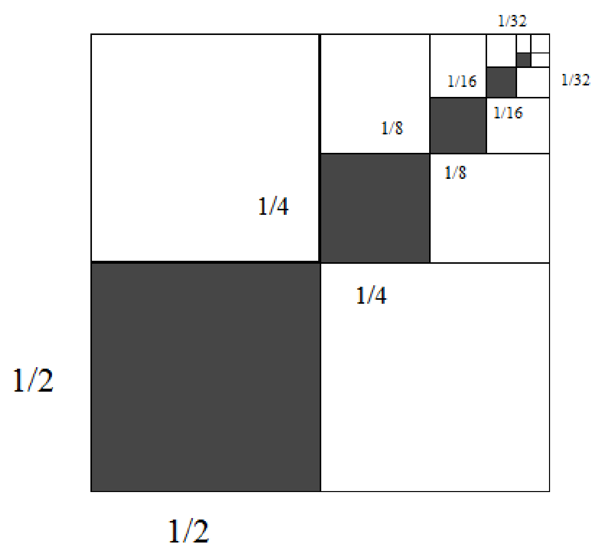

Figure 7. A "visual proof" of $\sum_{n=1}^{\infty}(\frac{1}{2})^{2n} = \frac{1}{3}$ [**4**, p. 899]

Visual representations have also been used to determine patterns in numbers. During the 17th century, Leibniz had asked the Bernoulli brothers if there might be a pattern in the binary expansion of π. For over 300 years, his question remained unanswered as the numbers in the expansion appear to be completely random. However, by generating images from the expansions, visually generated "randomness" has emerged [**4**, p. 901]. Mathematicians are now identifying patterns where none has so far been seen, as in this case in the expansions of irrational numbers [**4**, p. 901].

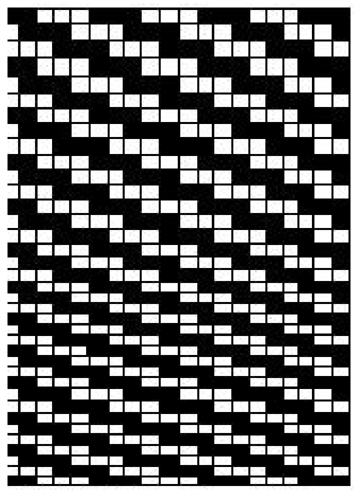

Figure 8. The first 1600 digits of $\frac{22}{7}\, mod\ 2$ [**4**, p. 902]

Borwein and Jorgensen have also determined visual structures are found in the coefficients of sequences of polynomials, and the existence of quasi-rational numbers, certain well known irrational numbers whose images appear suspiciously rational [**4**, p. 904].

VOCABULARY IN SECONDARY MATHEMATICS

Mathematical vocabulary is necessary when discussing mathematics in higher thinking settings. In 2000, the National Council of Teachers of Mathematics (NCTM) published *Principles and Standards for School Mathematics* which states that students need to be able to communicate about mathematics. Specifically, this means mathematics students must be able to understand mathematical information that is read, as well as be able to communicate with others both orally and in writing about mathematical ideas [**7**, p. 33]. With such importance given to vocabulary in mathematics, it's no wonder strategies to improve students' mathematical vocabulary have been used in secondary mathematics courses starting from Algebra continuing through upper division mathematics courses in universities. Visual word association, which has students learn the definition as well as a picture associated with the word, is being taught to students beginning in the middle grades [**7**, p. 36]. It can easily be transferred to higher level mathematical vocabulary for students in higher level thinking classes.

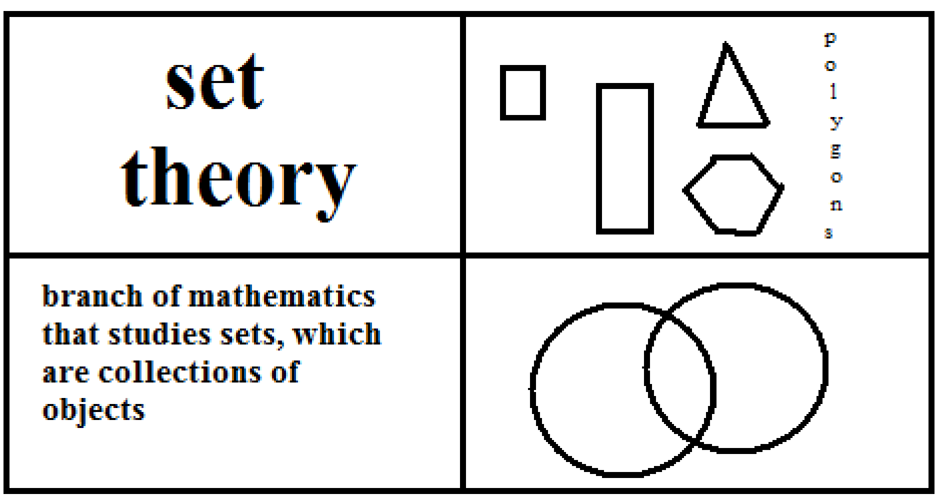

Figure 9. Visual Word Association Diagram [**7**, p. 35]

Gough reminds us that in teaching mathematics, we must surround the focal formal content language which is mathematics, with other informal instructional language. In other words, we not only need to learn how to “speak and do mathematics,” within mathematics as a language, but we also must learn how to talk in non mathematical language about what we are saying and doing in mathematics [**9**, p. 54]. This is essentially what vocabulary strategies in secondary mathematics courses build up to, having the individual become able to translate back and forth from mathematics and conversational language. Furthermore, students must learn to “encapsulate” and “*unitise*” the key ideas of any new concept [**9**, p. 56]. By definition, “*unitisizing*” is the conceptual act of forming a single new whole “thing” from what had previously been a collection of discrete unitary elements [**9**, p. 56]. In essence, as secondary mathematics teachers combine similar topics into units, students must be able to combine concepts into units as well, and connect them with mathematical vocabulary.

Chapter 3: Reasons for this Growth and Impact

Reasons for Growth

Why then are visuals and vocabulary instruction in mathematics and mathematics courses so important? As Hoag would say, of the many reasons for the use of visual material in teaching geometry, or mathematics in general, one of the most obvious would be to give meaning to mathematical abstractions. Furthermore, it makes the mathematics classroom more interesting and allows the students to see practical application of mathematics [**11**, p. 159]. During the Second World War, courses in "academic mathematics" were labeled too hard and did not meet the "immediate needs" of the adolescent population [**2**, p. 153]. Even today, many feel that any mathematical skill required in the job force can easily be "picked up on the job" and does not need to be drawn out in difficult mathematics classes. However, since some are of the opinion that the United States has fallen far behind the rest of the world in the war of mathematics and science, states such as Texas have adopted four year mathematics and science requirements for high school graduation in even their basic recommended graduation plans. Mathematicians have been presented with the task of making mathematics easier to understand, and more applicable to the people.

Coinciding with the desire for future generations to aid the United States' level of mathematical awareness, the illiteracy rate has seen a tremendous increase over the years. A study conducted by the United Nations Children's Fund predicted that the overall illiteracy rate would increase in the 21st century because only a fourth of the world's children were in school at the end of the 20th century [17, p. 1]. A more visual approach to mathematics is now not just a trend, but a necessity. Teaching "mathematics speak" is a attribute that makes men and women more desired in the workplace, and aids the next generation to desire an increase in mathematics education. While some feel that the "computer can do it all," a basic knowledge of problem solving skills and higher order thinking is more than handy when faced with the trials and tribulations of daily life. An increase in English Language Learners, as well, lends itself to the need for more vocabulary and visual instruction in order to bridge the communication gap.

IMPACT

Do the vocabulary and visual strategies improve the mathematical skills of students and mathematicians? Much research has been done in the last few years to warrant the use of visual "reminders" and vocabulary building. In an action research project conducted by Blessman and Myszczack, students created journals, mathematics dictionaries, student created literature, and graphic organizers to be evaluated for effectiveness on the comprehension of the students' mathematical concepts. In order to determine students' growth and success, questionnaires, surveys, and vocabulary checklists were administered to the students and teachers before and after the above mentioned strategies were implemented [**3**, p. 50]. As suggested by Harmon, both the teachers and the students designed the visual aids and graphic organizers that were used during the 6 week period. Based on the results, the strategies appeared to have had a positive effect on student comprehension of mathematics vocabulary words. The percentage of students exhibiting a high understanding of the mathematical concepts and vocabulary increased, while the percentage of students exhibiting a low or no understanding of the mathematical concepts and vocabulary decreased. Furthermore, all students were noted as appearing to increase their knowledge of mathematical vocabulary on daily assignments, mathematics journals, written explanations of mathematical problems, and class discussion [**3**, p. 55].

This result was mirrored by Schwarz who conducted research on a group of students who were found to be lacking understanding of content vocabulary, and therefore had poor communication skills that lead to unsuccessful communication of their mathematical understanding of a word problem and its answer [**15**, p. 1]. Vocabulary journals, mathematics journals, a mathematics word wall, and Multiple Intelligences strategies were implemented in order to address the students' deficiencies. Post test data revealed all students who participated in the study showed an increase in the use and understanding of mathematical vocabulary, as well as in communication of mathematical issues [**15**, p. 1].

While the use of mathematical vocabulary strategies have repeatedly shown to increase the mathematical mind of those exposed to the interventions, research also shows the same results in the use of visual strategies. In a research study conducted by Monroe, it was determined that the effective use of graphic organizers can help develop conceptual understanding of mathematics by promoting student involvement and emphasizing deep processing of words [**13**, p. 1]. The graphic organizers served as retrieval cues for information and helped to facilitate higher level thinking.

Conclusions

If society is to fix the problems of the past, there must be a fundamental reexamination of our basic objectives and a rebuilding of the entire educational system utilizing these growing strategies of visual mathematics and vocabulary strategies [**2**, p. 154]. Blessman and Myszczack would instruct teachers to focus their energies on bringing all students to a greater awareness of the language of mathematics [**3**, p. 56]. This includes the growing number of English Language Learners in mathematics classrooms. As the research presented has suggested, the use of visual mathematics and "mathematical language" can increase the mathematical understanding of those presented with the strategies, regardless of background or experience. Furthermore, the knowledge and understanding of the mathematical concepts are retained by the individuals longer, which is the goal of any educator. Overall, the research is available, along with the visual mathematics and vocabulary strategies, to enhance the mathematical ability of students of secondary mathematics and beyond.

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Vita

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